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# Isopiestic Determination of the Activity Coefficients of Some Aqueous Rare-Earth Electrolyte Solutions at $25{ }^{\circ} \mathrm{C} .5 . \mathrm{Dy}\left(\mathrm{NO}_{3}\right)_{3}$, $\mathrm{Ho}\left(\mathrm{NO}_{3}\right)_{3}$, and $\mathrm{Lu}\left(\mathrm{NO}_{3}\right)_{3}$ 

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The osmotic coetflcients of aqueous $\mathrm{Dy}\left(\mathrm{NO}_{3}\right)_{3}, \mathrm{Ho}\left(\mathrm{NO}_{3}\right)_{3}$, and $\mathrm{Lu}\left(\mathrm{NO}_{3}\right)_{3}$ have been measured from 0.18 to 6.5-7.2 $\mathrm{mol} \mathrm{kg}{ }^{-1}$ at $25{ }^{\circ} \mathrm{C}$ with the isoplestic method. The resulting osmotic coefficients were fltted to least-squares equations, which were used to calculate water activilies and mean molal activity coefficlents. These data are compared to activity data for other rare-earth nitrate solutions. The lower concentration results were also fitted to Plizer's equation.

## Introduction

Activity data have been published for 14 rare-earth chlorides, 12 rare-earth perchlorates, and 9 rare-earth nitrates at $25^{\circ} \mathrm{C}$ (1-4). The water activities of the rare-earth chlorides and perchlorates have been correlated with trends in cation hydration resulting from the lanthanide contraction (2). However, inner and outer sphere complexes form in rare-earth nitrate solutions, and these complexes extensively modify the hydration trends of the rare-earth cations (5).

In this paper isopiestic data are reported for aqueous Dy$\left(\mathrm{NO}_{3}\right)_{3}, \mathrm{Ho}\left(\mathrm{NO}_{3}\right)_{3}$, and $\mathrm{Lu}\left(\mathrm{NO}_{3}\right)_{3}$ solutions. These data provide additional information about the modification of cation hydration by complex formation, and about changes in the amount of complex formation with changes in the ionic radii of the rare earths.

## Experimental Section

The experimental details are nearly identical with those of the previous study (4). The measurements were made at 25.00 $\pm 0.005{ }^{\circ} \mathrm{C}$ (IPTS-68). Isoplestic equilibration times were 5-36 days, with the longer times used for lower concentrations. The isopiestic standards were $\mathrm{CaCl}_{2}$ stock no. 1 and KCl ; their analyses are described elsewhere (6). All weights were converted to mass. The molecular masses used were 348.515 g $\mathrm{mol}^{-1}$ for $\mathrm{Dy}\left(\mathrm{NO}_{3}\right)_{3}, 350.945 \mathrm{~g} \mathrm{~mol}^{-1}$ for $\mathrm{Ho}\left(\mathrm{NO}_{3}\right)_{3}, 360.985 \mathrm{~g}$ $\mathrm{mol}^{-1}$ for $\mathrm{Lu}\left(\mathrm{NO}_{3}\right)_{3}, 110.986 \mathrm{~g} \mathrm{~mol}^{-1}$ for $\mathrm{CaCl}_{2}$, and 74.551 g $\mathrm{mol}^{-1}$ for KCl.

The rare-earth nitrate solutions were prepared from pure rare-earth oxides and nitric acid and were adjusted to their

Table I. Isopiestic Molalities of Some Rare-Earth Nitrate Solutions from Measurements with KCl Reference Solutions

| $\left[\mathrm{Dy}\left(\mathrm{NO}_{3}\right)_{3}\right]$, <br> $m$ | $\left[\mathrm{Ho}\left(\mathrm{NO}_{3}\right)_{3}\right]$, <br> $m$ | $\left[\mathrm{Lu}\left(\mathrm{NO}_{3}\right)_{3}\right]$, <br> $m$ | $[\mathrm{KCl}], m$ | $\Phi(\mathrm{KCl})$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.18530 | 0.18387 | 0.18152 | 0.31905 | 0.9047 |
| 0.19247 | 0.19102 | 0.18851 | 0.33217 | 0.9041 |
| 0.30176 | 0.29855 | 0.29308 | 0.53764 | 0.8986 |
| 0.31518 | 0.31175 | 0.30590 | 0.56410 | 0.8982 |
| 0.41371 | 0.40873 | 0.39934 | 0.76297 | 0.8968 |
| 0.51795 | 0.51046 | 0.49803 | 0.98476 | 0.8972 |
| 0.52721 | 0.51943 | 0.50610 | 1.0054 | 0.8974 |
| 0.59709 | 0.58811 | 0.57170 | 1.1613 | 0.8985 |
| 0.67769 | 0.66663 | 0.64644 | 1.3459 | 0.9005 |
| 0.75573 | 0.74280 | 0.71842 | 1.5324 | 0.9031 |
| 0.84150 | 0.82677 | 0.79655 | 1.7424 | 0.9065 |
| 0.93624 | 0.91868 | 0.88253 | 1.9811 | 0.9110 |
| 1.0457 | 1.0254 | 0.98131 | 2.2650 | 0.9169 |
| 1.0689 | 1.0492 | 1.0034 | 2.3282 | 0.9183 |

equivalence concentrations. The stock solutions were analyzed by EDTA and the gravimetric sulfate method, and the concentrations were measured to at least $0.1 \%$.
Duplicate samples were used in the equilibrations. The average molalities are known to at least $\pm 0.1 \%$ above 0.3 mol $\mathrm{kg}^{-1}$ and better than $0.15 \%$ at lower concentrations. However, most equilibrations were to $\pm 0.05 \%$ or better. The molalities at isopiestic equilibrium are reported in Tables I and II.
The higher rare-earth nitrate concentrations are for supersaturated solutions (5). Dy $\left(\mathrm{NO}_{3}\right)_{3}$ was the only salt that had crystallization problems at the highest concentrations; consequently two of its points are based on single samples. After each high-concentration equilibration, the isopiestic cups were carefully examined to verity the absence of crystals.
The presence of a small amount of nitrite ions in the rareearth nitrate solutions was indicated by a positive "brown ring" test. The rare-өarth nitrate solutions were heated during their preparation from rare-earth oxides and nitric acid. Thus, any nitrite or nitrogen oxides in the nitric acid should have been decomposed or expelled. Consequently, any nitrite in the final solution was formed after preparation.

A $4.4551 \mathrm{~mol} \mathrm{~kg}^{-1} \mathrm{La}\left(\mathrm{NO}_{3}\right)_{3}$ solution, prepared several years ago, was studied further. Each nitrate ion decomposing to nitrite consumes two hydrogen ions, so pH changes will result. The

Table II. Isopiestic Molalities of Some Rare-Earth Nitrate Solutions from Measurements with $\mathrm{CaCl}_{2}$ Reference Solutions

${ }^{a}$ These two $\mathrm{Dy}\left(\mathrm{NO}_{3}\right)_{3}$ molalities are for single samples. All other concentrations in Tables I and II are the average of duplicate samples.


Figure 1. Differences between experimental and calculated osmotic coefficients of rare-earth nitrate solutions at $25^{\circ} \mathrm{C}$ : ( ${ }^{\circ}$ ) isopiestic vs. $\mathrm{CaCl}{ }_{2}$; $(O)$ isoplestic vs. $\mathrm{KCl} ;( \rangle)$ estimated from electrical conductances.
solution pH was found to be 3.30 ; titration with dilute $\mathrm{HNO}_{3}$ indicates that the correct equivalence pH should be 3.0. If slow cation hydrolysis is negligible, and the hydrogen ion activity equated to its concentration, then there is $0.002 \% \mathrm{La}\left(\mathrm{NO}_{2}\right)_{3}$ and $99.998 \% \mathrm{La}\left(\mathrm{NO}_{3}\right)_{3}$ in the solution.

As a further check, a sample of this $\mathrm{La}\left(\mathrm{NO}_{3}\right)_{3}$ was acidified with $\mathrm{HClO}_{4}$ and then oxidized with $\mathrm{KMnO}_{4}$ in the absence of light. This method indicated a maximum of $0.0017 \% \mathrm{La}\left(\mathrm{NO}_{2}\right)_{3}$, which is in reasonable agreement with the pH-based estimate. Nitrate decomposition is therefore too small to have a detect-

Table III. Coefficients and Powers for Osmotic Coefficient Polynomials

| $i$ | $r_{i}$ | $A_{i}\left(\mathrm{Dy}\left(\mathrm{NO}_{3}\right)_{3}\right)$ | $A_{i}\left(\mathrm{Ho}\left(\mathrm{NO}_{3}\right)_{3}\right)$ | $A_{i}\left(\mathrm{Lu}\left(\mathrm{NO}_{3}\right)_{3}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.75 | -2.009692 | -2.542587 | -3.680921 |
| 2 | 1.00 | 38.25539 | 42.05577 | 49.52778 |
| 3 | 1.25 | -91.96541 | -102.3766 | -120.3688 |
| 4 | 1.50 | 106.1766 | 120.76267 | 141.6557 |
| 5 | 1.75 | -66.26518 | -77.26793 | -89.40156 |
| 6 | 2.00 | 21.54498 | 25.78285 | 29.03241 |
| 7 | 2.25 | -2.873645 | -3.527294 | -3.817119 |
| SD |  | 0.0013 | 0.0013 | 0.0016 |

able effect on the properties of rare-earth nitrate solutions.

## Calculations and Errors

The osmotic coefficients, $\Phi$, of the rare-earth nitrate solutions were calculated from eq 1

$$
\begin{equation*}
\Phi=\nu^{*} m^{*} \Phi^{*} / \nu m \tag{1}
\end{equation*}
$$

where $m$ is the rare-earth nitrate solution molality and $\nu=4$ is the number of ions formed by the complete dissociation of one molecule of rare-earth nitrate. The equivalent quantities for the KCl or $\mathrm{CaCl}_{2}$ isopiestic standards are indicated with asterisks. The osmotic coefficients of the reference solutions were calculated from available equations (7,8). These $\Phi^{*}$ values are reported in Tables I and II along with the isopiestic molalities.

The osmotic coefficients of the rare-earth nitrates were represented by eq 2

$$
\begin{equation*}
\Phi=1-(A / 3) m^{1 / 2}+\sum_{l} A_{l} m_{l} \tag{2}
\end{equation*}
$$

where $A=8.6430$ is the Debye-Hückel limiting slope for 3-1 electrolytes. The mean molal activity coefficients are then given by eq 3

$$
\begin{equation*}
\ln \gamma_{ \pm}=-A m^{1 / 2}+\sum_{l} A_{l}\left[\left(r_{l}+1\right) / r_{l}\right] m_{l} \tag{3}
\end{equation*}
$$

which is the Debye-Hückel limiting law plus a series in the molal concentration. Series in $m^{1 / 4}$ worked best for these salts. Water activities can be calculated from eq 4

$$
\begin{equation*}
\ln a_{1}=-\nu m M_{1} \Phi / 1000 \tag{4}
\end{equation*}
$$

where $M_{1}=18.0154 \mathrm{~g} \mathrm{~mol}^{-1}$ is the molecular mass of water.
All of the isopiestic data in Tables I and II were given unit weights for the least-squares fits to eq 2. Dilute solution osmotic coefficients were estimated from ion-size parameters derived from electrical-conductance data (3), and these $\Phi$ values were used to constrain eq 2 and 3 at concentrations below the isopiestic measurements. These estimated osmotic coefficients, which are probably uncertain by 0.005 units, were also given unit weights. The least-squares parameters to eq 2 and 3 are given in Table III, along with the standard deviations for the best fits. Values of $\Phi, a_{1}$, and $\gamma_{ \pm}$, for various round concentrations are reported in Table IV.

The osmotic coefficients up to $2.0 \mathrm{~mol} \mathrm{~kg}^{-1}$ were also fitted to Pitzer's equation (9), and the parameters are listed in Table V. Parameters for other rare-earth electrolytes are given elsewhere (4,9). Pitzer's equation does not represent the data as well as eq 2 since it involves fewer parameters.

The maximum uncertainties in the experimental $\Phi$ values are $0.3 \%$, with the probable error being $0.2 \%$. The differences between the experimental $\Phi$ values and eq 2 are shown in Figure 1.

Isopiestic data for rare-earth nitrates seem to be fairly reproducible. Libuś et al. (10) have reported data for three rare-earth nitrates at $25^{\circ} \mathrm{C}$. Their results for $\mathrm{Nd}\left(\mathrm{NO}_{3}\right)_{3}$ and $\mathrm{Sm}\left(\mathrm{NO}_{3}\right)_{3}$ at all concentrations, and for $\mathrm{Gd}\left(\mathrm{NO}_{3}\right)_{3}$ up to 2.0 mol $\mathrm{kg}^{-1}$, agree with previous data from our laboratories to within


Figure 2. Mean molal activity coefficients of rare-aarth nitrate solutions at constant molalities.
experimental error (3, 4). However, differences for $\mathrm{Gd}\left(\mathrm{NO}_{3}\right)_{3}$ at higher concentrations are significant.

## Results and Discussion

Figures 2 and 3 show the mean molal activity coefficients and water activities of the rare-earth nitrates, at various molalities, as a function of the cation radius (11). The activity coefficients at low concentrations decrease from $\mathrm{La}\left(\mathrm{NO}_{3}\right)_{3}$ to $\mathrm{Sm}\left(\mathrm{NO}_{3}\right)_{3}\left(\mathrm{Eu}\left(\mathrm{NO}_{3}\right)_{3}\right.$ was not studied) and then increase to $\mathrm{Lu}-$ $\left(\mathrm{NO}_{3}\right)_{3}$. By $1.0-1.2 \mathrm{~mol} \mathrm{~kg}{ }^{-1}, \gamma_{ \pm}$is nearly constant for $\mathrm{La}\left(\mathrm{NO}_{3}\right)_{3}$ to $\mathrm{Nd}\left(\mathrm{NO}_{3}\right)_{3}$ and then increases to $\mathrm{Lu}\left(\mathrm{NO}_{3}\right)_{3}$. The water activities

Table IV. Osmotic Coefficients, Water Activities, and Activity Coefficients at Even Molalities


Table V. Parameters for Pitzer's Equation

|  | parameteI | $\mathrm{Dy}\left(\mathrm{NO}_{3}\right)_{3}$ | $\mathrm{Ho}\left(\mathrm{NO}_{3}\right)_{3}$ |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{Lu}\left(\mathrm{NO}_{3}\right)_{3}$ |  |  |
| $3 \beta^{(0)} / 2$ | 0.8484 | 0.8769 | 0.9264 |
|  | $3 \beta^{(1)} / 2$ | 7.700 | 7.700 |
| SD | $\left(3^{3 / 2} / 2\right) \mathrm{C}^{\Phi}$ | -0.1809 | -0.1852 |
| S |  | 0.0085 | -0.700 |
|  |  | 0.0086 | 0.0080 |

exhibit similar trends but in the opposite direction.
In contrast, the activity data for rare-earth chlorides and perchlorates (1,2) are S-shaped as a function of the cation
radius. Complex formation is fairly weak for these two systems, so activity trends mainly reflect cation hydration. An increase in total hydration occurs from $\mathrm{La}^{3+}$ to $\mathrm{Lu}^{3+}$, due to the increase in the surface charge density of the bare ion as the lanthanide contraction occurs. Superimposed on this is an inner sphere hydration number decrease between $\mathrm{Nd}^{3+}$ and $\mathrm{Tb}^{3+}$. This inner sphere hydration decrease causes the inner sphere (hydrated) radius to decrease more rapidly (12), and total hydration to increase more rapidly, with ionic radius in the middle of the rare-earth series. The same model was used to explain the


Figure 3. Water activities of rare-earth nitrate solutions at constant molallities.
variation of the standard state ionic entropies (13).
Libuś et al. (10) suggested that an inner sphere increase, rather than a decrease, occurs with decreasing ionic radius. This conflicts with X-ray diffraction results (12) which show the inner sphere hydration number decreases by one between $\mathrm{Nd}^{3+}$ and $\mathrm{Tb}^{3+}$. Also, hydration numbers obtained by fitting thermodynamic models to volumetric and heat-capacity data (14) indicate a unit decrease in the cation hydration number. Glass
transition temperatures for the rare-earth chlorides and perchlorates likewise correlate with hydration trends (15).

Both Inner and outer sphere complexes form in rare-earth nitrate solutions, and they begin to modify rare-earth hydration at falry low concentrations by dlsplacing water from the cation hydration sheath. Flgure 2 indicates that $\gamma_{ \pm}$decreases from $\mathrm{La}\left(\mathrm{NO}_{3}\right)_{3}$ to $\mathrm{Sm}\left(\mathrm{NO}_{3}\right)_{3}$ or $\mathrm{Eu}\left(\mathrm{NO}_{3}\right)_{3}$ and then Increases to $\mathrm{Lu}(\mathrm{N}$ $\left.\mathrm{O}_{3}\right)_{3}$ at low concentrations. Above ca. $1.0-1.2 \mathrm{~mol} \mathrm{~kg}^{-1}$ the minimum disappears, and $\gamma_{ \pm}$Increased from $\mathrm{La}\left(\mathrm{NO}_{3}\right)_{3}$ to $\mathrm{Lu}-$ $\left(\mathrm{NO}_{3}\right)_{3}$. This suggests that complex formation increases from $\mathrm{La}\left(\mathrm{NO}_{3}\right)_{3}$ to $\mathrm{Sm}\left(\mathrm{NO}_{3}\right)_{3}-\mathrm{Eu}\left(\mathrm{NO}_{3}\right)_{3}$ and then decreases to $\mathrm{Lu}\left(\mathrm{NO}_{3}\right)_{3}$ at low concentrations. At higher concentrations, complex formation should decrease from $\mathrm{La}\left(\mathrm{NO}_{3}\right)_{3}$ to $\mathrm{Lu}\left(\mathrm{NO}_{3}\right)_{3}$. This is in complete agreement with changes in the electrical-conductance data (5) and with the known association constants (16) at low concentrations.

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## Glossary

$\Phi$
$\nu \quad$ number of lons formed by the dissociation of one molecule of solute
$m \quad$ molal concentration of the solute, mol $\mathrm{kg}^{-1}$
$\gamma_{ \pm} \quad$ mean molal activity coefficient
$a_{1} \quad$ water activity
$\beta^{(0)}$, parameters for Pitzer's equation
-
A
$A_{i}$
$r_{i}$
symbols with asterisks refer to KCl or $\mathrm{CaCl}_{2}$ isopiestic standards
Debye-Hückel constant
least-squares coefficients of eq 2 and 3

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